Data Fusion of Dual Foot-Mounted Zero Velocity Update (ZUPT) Aided Inertial Navigation Systems (INSs) using Centroid Method

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Notations

- Define the navigation state vector of right foot INS at time instant $k \in \mathbb{N}^+$

$$\hat{\mathbf{x}}_k^{(R)} \triangleq \begin{bmatrix} \hat{\mathbf{p}}_k^{(R)} & \hat{\mathbf{v}}_k^{(R)} & \hat{\boldsymbol{\theta}}_k^{(R)} \end{bmatrix}^T$$

  - $\hat{\mathbf{p}}_k^{(R)} \in \mathbb{R}^3$ is the position estimate of right foot INS
  - $\hat{\mathbf{v}}_k^{(R)} \in \mathbb{R}^3$ is the velocity estimate of right foot INS
  - $\hat{\boldsymbol{\theta}}_k^{(R)} \in \mathbb{R}^3$ is the attitude estimate of right foot INS

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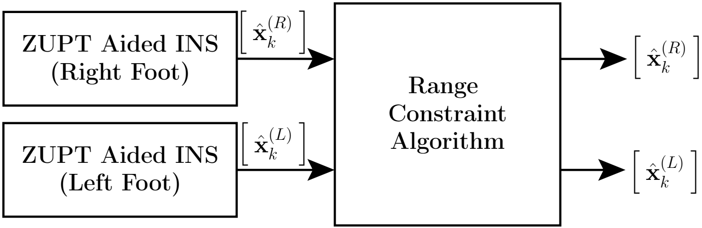
- Define the navigation state vector of left foot INS at time instant $k \in N^+$

$$\hat{\mathbf{x}}_k^{(L)} \triangleq \begin{bmatrix} \hat{\mathbf{p}}_k^{(L)} & \hat{\mathbf{v}}_k^{(L)} & \hat{\boldsymbol{\theta}}_k^{(L)} \end{bmatrix}^T$$

  - $\hat{\mathbf{p}}_k^{(L)} \in \mathbb{R}^3$ is the position estimate of left foot INS
  - $\hat{\mathbf{v}}_k^{(L)} \in \mathbb{R}^3$ is the velocity estimate of left foot INS
  - $\hat{\boldsymbol{\theta}}_k^{(L)} \in \mathbb{R}^3$ is the attitude estimate of left foot INS

Range Constraint Algorithm



At any instant of time $k \in \mathbb{N}^+$, the distance between position estimates of right and left foot INS cannot be greater than a parameter known as *foot-to-foot maximum separation* (γ). Based on this range constraint, we develop Centroid Method to fuse the navigation information of right and left foot-mounted ZUPT aided INS.

Centroid Method - Problem Formulation

- Define the unconstrained joint position estimate vector at time instant $k \in \mathbb{N}^+$

$$\hat{\mathbf{p}}_k^{(joint)} \triangleq \begin{bmatrix} \hat{\mathbf{p}}_k^{(R)} & \hat{\mathbf{p}}_k^{(L)} \end{bmatrix}^T$$

  - $\hat{\mathbf{p}}_k^{(R)} \in \mathbb{R}^3$ is the position estimate of right foot INS
  - $\hat{\mathbf{p}}_k^{(L)} \in \mathbb{R}^3$ is the position estimate of left foot INS

Centroid Method - Problem Formulation

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  - $\hat{\mathbf{p}}_k^{(L)} \in \mathbb{R}^3$ is the position estimate of left foot INS

- Define the matrix

$$\mathbf{L} \triangleq \begin{bmatrix} \mathbf{I}_3 & -\mathbf{I}_3 \end{bmatrix}$$

  where $\mathbf{I}_q$ is the identity matrix of size q.

Centroid Method - Problem Formulation

At any instant of time $k \in \mathbb{N}^+$, following condition must hold:

$$\left\|\mathbf{L}\hat{\mathbf{p}}_k^{(joint)}\right\|_2^2 = \left\|\hat{\mathbf{p}}_k^{(R)} - \hat{\mathbf{p}}_k^{(L)}\right\|_2^2 \leq \gamma^2$$

where $\gamma \in \mathbb{R}^+$ is *the foot-to-foot maximum separation*

Centroid Method - Problem Formulation

At any instant of time $k \in \mathbb{N}^+$, following condition must hold:

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where $\gamma \in \mathbb{R}^+$ is *the foot-to-foot maximum separation*

Hence, we can formulate the problem in constrained least squares framework as:

$$\mathbf{p}_k^{(joint)} = \underset{\mathbf{p} \in \mathbb{R}^6}{\text{argmin}} \left(\left|\left|\hat{\mathbf{p}}_k^{(joint)} - \mathbf{p}\right|\right|_2^2\right) \text{ subject to } \left|\left|\mathbf{L}\mathbf{p}_k^{(joint)}\right|\right|_2^2 \leq \gamma^2 \tag{1}$$

where

$$\mathbf{p}_k^{(joint)} \triangleq \begin{bmatrix}\mathbf{p}_k^{(R)} & \mathbf{p}_k^{(L)}\end{bmatrix}^T$$

is the range constrained joint position estimate vector at time instant $k \in \mathbb{N}^+$

Centroid Method - Solution

In Lagrangian formulation with Lagrange multiplier λ_C:

$$J\left(\mathbf{p}_k^{(joint)}, \lambda_C\right) \triangleq \left\|\hat{\mathbf{p}}_k^{(joint)} - \mathbf{p}_k^{(joint)}\right\|_2^2 + \lambda_C \, \Psi\left(\mathbf{p}_k^{(joint)}\right)$$

where

$$\Psi\left(\mathbf{p}_k^{(joint)}\right) = \left\|\mathbf{L}\mathbf{p}_k^{(joint)}\right\|_2^2 - \gamma^2$$

Centroid Method - Solution

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where

$$\Psi\left(\mathbf{p}_k^{(joint)}\right) = \left|\left|\mathbf{L}\mathbf{p}_k^{(joint)}\right|\right|_2^2 - \gamma^2$$

Lagrange condition (normal equations):

$$\frac{\partial J\left(\mathbf{p}_k^{(joint)}, \lambda_C\right)}{\partial \mathbf{p}_k^{(joint)}} = \mathbf{0} \implies \left(\mathbf{I}_6 + \lambda_C\, \mathbf{L}^T\mathbf{L}\right)\, \mathbf{p}_k^{(joint)} = \hat{\mathbf{p}}_k^{(joint)} \tag{2}$$

$$\frac{\partial J\left(\mathbf{p}_k^{(joint)}, \lambda_C\right)}{\partial \lambda_C} = 0 \implies \left|\left|\mathbf{L}\mathbf{p}_k^{(joint)}\right|\right|_2^2 = \gamma^2 \tag{3}$$

Centroid Method - Solution

The solution of equation (2) is

$$\mathbf{p}_k^{(joint)} = \left(\mathbf{I}_6 + \lambda_C \mathbf{L}^T \mathbf{L}\right)^{-1} \hat{\mathbf{p}}_k^{(joint)} \tag{4}$$

Centroid Method - Solution

The solution of equation (2) is

$$\mathbf{p}_k^{(joint)} \;=\; \left(\mathbf{I}_6 \;+\; \lambda_C \mathbf{L}^T \mathbf{L}\right)^{-1} \;\hat{\mathbf{p}}_k^{(joint)} \tag{4}$$

Solve for λ_C:

First note that

$$(\mathbf{I}_6 \;+\; \lambda_C \mathbf{L}^T \mathbf{L})^{-1} \;=\; \frac{1}{1 + 2\,\lambda_C} \begin{bmatrix} (1 + \lambda_C)\;\mathbf{I}_3 & \lambda_C\;\mathbf{I}_3 \\ \lambda_C\;\mathbf{I}_3 & (1 + \lambda_C)\;\mathbf{I}_3 \end{bmatrix} \tag{5}$$

provided $\lambda_C \;\neq\; -0.5$

Centroid Method - Solution

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provided $\lambda_C \neq -0.5$

Substitute Eq. (4) in Eq. (3)

$$\Longrightarrow \left\| L\ \left(\mathbf{I}_6 + \lambda_C \mathbf{L}^T \mathbf{L}\right)^{-1} \hat{\mathbf{p}}_k^{(joint)} \right\|_2^2 = \gamma^2$$

Centroid Method - Solution

Substitute Eq. (5) and simplifying:

$$\lambda_C = 0.5 \left(\frac{\hat{d}_k}{\gamma} - 1 \right) \tag{6}$$

where

$$\hat{d}_k \triangleq \left\| \hat{\mathbf{p}}_k^{(R)} - \hat{\mathbf{p}}_k^{(L)} \right\|_2$$

is the distance between the unconstrained position estimates of right and left foot INS.

Centroid Method - Solution

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$$\lambda_C = 0.5 \left(\frac{\hat{d}_k}{\gamma} - 1 \right) \tag{6}$$

where

$$\hat{d}_k \triangleq \left\| \hat{\mathbf{p}}_k^{(R)} - \hat{\mathbf{p}}_k^{(L)} \right\|_2$$

is the distance between the unconstrained position estimates of right and left foot INS.

Substitute Eq. (5) , Eq. (6) in Eq. (4) and simplifying:

$$\mathbf{p}_k^{(R)} = \frac{\left(\hat{d}_k + \gamma\right) \hat{\mathbf{p}}_k^{(R)} + \left(\hat{d}_k - \gamma\right) \hat{\mathbf{p}}_k^{(L)}}{2\, \hat{d}_k} \tag{7}$$

$$\mathbf{p}_k^{(L)} = \frac{\left(\hat{d}_k + \gamma\right) \hat{\mathbf{p}}_k^{(L)} + \left(\hat{d}_k - \gamma\right) \hat{\mathbf{p}}_k^{(R)}}{2\, \hat{d}_k} \tag{8}$$

Graphical Representation

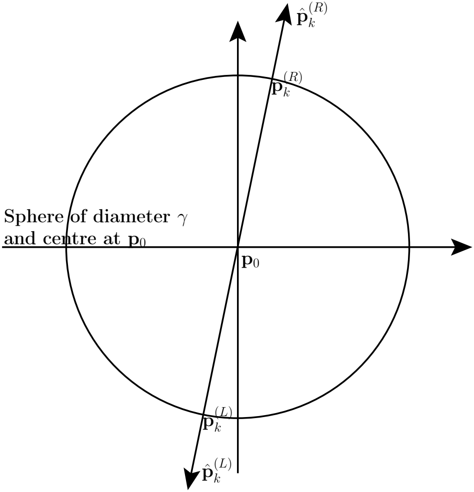

Figure: Graphical representation of unconstrained and constrained position estimates

Alternate Problem Formulation

In the figure, the centre of the sphere

$$\mathbf{p}_0 = \frac{\mathbf{p}_k^{(R)} + \mathbf{p}_k^{(L)}}{2} \tag{9}$$

Hence, by substituting

$$\mathbf{p}_k^{(L)} = 2\,\mathbf{p}_0 - \mathbf{p}_k^{(R)} \tag{10}$$

an alternate formulation of (1) is written as

$$\mathbf{p}_k^{(R)} = \underset{\mathbf{p} \in \mathbb{R}^3}{\text{argmin}} \left(\left|\left| \hat{\mathbf{p}}_k^{(joint)} - \mathbf{L}^T \mathbf{p} + 2\,\mathbf{p}_0^{(joint)} \right|\right|_2^2 \right)$$

$$\text{subject to } \left|\left| \mathbf{p}_k^{(R)} - \mathbf{p}_0 \right|\right|_2^2 \leq \frac{\gamma^2}{4}$$

where

$$\mathbf{p}_0^{(joint)} \triangleq \begin{bmatrix} \mathbf{0} & \mathbf{p}_0 \end{bmatrix}^T$$

Alternate Problem Formulation

Solution in Lagrangian framework is found to be:

$$\mathbf{p}_k^{(R)} = \frac{\gamma\,\hat{\mathbf{p}}_k^{(R)} - \gamma\,\hat{\mathbf{p}}_k^{(L)} + 2\,\hat{d}_k\,\mathbf{p}_0}{2\,\hat{d}_k} \tag{11}$$

Substitute Eq. (11) in Eq. (10)

$$\mathbf{p}_k^{(L)} = \frac{\gamma\,\hat{\mathbf{p}}_k^{(L)} - \gamma\,\hat{\mathbf{p}}_k^{(R)} + 2\,\hat{d}_k\,\mathbf{p}_0}{2\,\hat{d}_k} \tag{12}$$

If we define the centroid

$$\mathbf{p}_0 \triangleq \frac{\hat{\mathbf{p}}_k^{(R)} + \hat{\mathbf{p}}_k^{(L)}}{2} \tag{13}$$

solution given by Eq. (11) and (12) reduces to the solution given by Eq. (7) and Eq. (8) respectively.

Weighted Centroid Method

- Instead of defining the centroid as the midpoint of right and left foot INS position estimates as in Eq. (13), we can make use of the entries of error covariance matrix $\mathbf{P}$, to define the weighted centroid.
- Range constrained right and left foot position estimates given by Eq. (11) and (12) is used as the pseudo-measurement in complementary feedback Kalman Filter.

Centroid Method - Algorithm Description

If $\hat{d}_k > \gamma$, then execute the following steps.

- Calculate the pseudo-measurement for the position given by Eq. (11) and Eq. (12), where $\mathbf{p}_0$ is given by Eq. (13) or as explained in weighted centroid method.

Centroid Method - Algorithm Description

If $\hat{d}_k > \gamma$, then execute the following steps.

- Calculate the pseudo-measurement for the position given by Eq. (11) and Eq. (12), where $\mathbf{p}_0$ is given by Eq. (13) or as explained in weighted centroid method.
- Kalman filter position pseudo-measurement update for right foot INS:
  1. Compute the Kalman gain:
  $$\mathbf{K}_k^{(R)} = \mathbf{P}_k^{(R)} \left(\mathbf{H}_{(pos)}^{(R)}\right)^T \left[\mathbf{H}_{(pos)}^{(R)} \mathbf{P}_k^{(R)} \left(\mathbf{H}_{(pos)}^{(R)}\right)^T + \mathbf{R}_{(pos)}^{(R)}\right]^{-1}$$
  2. Correct the navigation state vector using the position pseudo-measurement:
  $$\hat{\mathbf{x}}_k^{(R)} = \hat{\mathbf{x}}_k^{(R)} + \mathbf{K}_k^{(R)} \left[\mathbf{p}_k^{(R)} - \mathbf{H}_{(pos)}^{(R)} \hat{\mathbf{x}}_k^{(R)}\right]$$
  3. Correct the state covariance matrix:
  $$\mathbf{P}_k^{(R)} = \left(\mathbf{I} - \mathbf{K}_k^{(R)} \mathbf{H}_{(pos)}^{(R)}\right) \mathbf{P}_k^{(R)}$$

Centroid Method - Algorithm Description

- Kalman filter position pseudo-measurement update for left foot INS:
  1. Compute the Kalman gain:

     $$\mathbf{K}_k^{(L)} = \mathbf{P}_k^{(L)} \left(\mathbf{H}_{(pos)}^{(L)}\right)^T \left[\mathbf{H}_{(pos)}^{(L)} \mathbf{P}_k^{(L)} \left(\mathbf{H}_{(pos)}^{(L)}\right)^T + \mathbf{R}_{(pos)}^{(L)}\right]^{-1}$$

     where $\mathbf{H}_{(pos)}^{(L)} = \begin{bmatrix}\mathbf{I}_3 & \mathbf{0}_{3\times 6}\end{bmatrix}$ is the position pseudo-measurement observation matrix of left foot INS and $\mathbf{R}_{(pos)}^{(L)}$ is the position pseudo-measurement noise covariance matrix of left foot INS.
  2. Correct the navigation state vector using the position pseudo-measurement:

     $$\hat{\mathbf{x}}_k^{(L)} = \hat{\mathbf{x}}_k^{(L)} + \mathbf{K}_k^{(L)} \left[\mathbf{p}_k^{(L)} - \mathbf{H}_{(pos)}^{(L)} \hat{\mathbf{x}}_k^{(L)}\right]$$

  3. Correct the state covariance matrix:

     $$\mathbf{P}_k^{(L)} = \left(\mathbf{I} - \mathbf{K}_k^{(L)} \mathbf{H}_{(pos)}^{(L)}\right) \mathbf{P}_k^{(L)}$$

Thank You